Department of Mathematics First Incomplete Examination Duration: 75 minutes

Answer all of the following questions.

Calculators and mobile phones are not allowed.

1. Find the area of the region bounded by the graphs of the functions:

$$y=x^2+4x,\,y=x.$$

5 points

2. The region R is bounded by the graphs of the curves xy = 2, 2y + 3x - 13 = 0. Sketch the region R and find the volume of the solid generated by revolving the region R about the line x = -1.

6 points

- 3. Find the arc length of the graph of $y = 6 2\sqrt{x^3}$ from (1,4) to (4,-10).
 6 points
- 4. Let $f(x) = \sqrt{e^{x+1} + 4}, x \in (-\infty, \infty)$.

(a) Show that f is one-to-one.

(b) Find $f^{-1}(x)$ and state the domain and range of f^{-1} .

2+5 points

- 5. (a) Let $f(x) = 5e^x 2e^{-x} 2$, $(x \in (-\infty, \infty))$. Find the slope of the tangent line to the graph of f^{-1} at the point (1,0).
 - (b) Find y' if

$$y = \frac{\sqrt[5]{\dot{x}^3 + 2x}(x^2 + 1)^6}{\sqrt[3]{1 - x^3}} + \ln|x^4 - e^x|.$$

4+4 points

6. Evaluate the following integrals

(a)
$$\int (\tan x - 1)^2 dx$$
 (b) $\int \frac{e^{2x} dx}{\sqrt{10 - e^{2x}}}$